SL Note 96–38 (AP)

Beam Size, Emittance and Optical Functions in LEP

John M. Jowett

22 May 1996

Abstract

This note has a twofold purpose. The first is to discuss the physical principles behind the estimation of emittance from beam size using measuring devices (such as the BEUV monitors) in LEP. This includes an explanation of some of the discrepancies between "measured emittances" and the luminosity in LEP.

The second, closely related, purpose is more technical. The program MAD calculates a variety of functions describing the optics of LEP. The relation of beam sizes to emittance is used to illustrate the proper application of MAD's capabilities and the meaning of the results. Some pitfalls in application of semi-intuitive recipes are pointed out.

Most of the plots in this note are better viewed in colour. The note is also available on the World-Wide-Web at:

`http://hpariel.cern.ch/jowett/papers/beuvbeat/beuvbeat.html`

Contents

List of Figures

List of Tables

1 Introduction

There are apparent inconsistencies between estimates of the LEP beam emittances from direct measurements via the BEUV monitors and the luminosity measured by the LEP experiments. While LEP ran at the Z-resonance, these were not a serious concern as the LEP experiments could measure luminosity rapidly and without any knowledge of the emittance. However the much smaller cross-sections at LEP2 energies will make it harder to measure luminosity directly and there will be a greater reliance on constructing the beam sizes at the IPs from the measured emittances.

As it has been shown that the discrepancies are unlikely to be instrumental [1], the current view is that the discrepancy arises from poor knowledge of the beam optics at the source point for the synchrotron light detected by the BEUV. In particular, it has been shown [2] that better agreement can be obtained by using measured β-functions and including the linear focusing perturbations due to the beam-beam interactions.

In this note[1], I shall examine another aspect of the question, namely: is the usual description of the beam optics in terms of β- and dispersion functions adequate to describe the relation

$$\sigma_x = \sqrt{\epsilon_x\beta_x + D_x\sigma_\varepsilon^2}, \qquad \sigma_y = \sqrt{\epsilon_y\beta_y + D_y\sigma_\varepsilon^2} \tag{1}$$

between the emittance and beam size? Given the strong energy-sawtoothing effects at high energy, we might need to re-examine this relationship. In particular, we should review how we get the values of $\beta_{x,y}$ and $D_{x,y}$ and how much they depend on the (unknown) errors in the machine.

It has also been shown, in simulation [3], that it is possible, at least in principle, to make the β-functions correspond more approximately to their theoretical values by a matching procedure involving the strengths of the interaction region quadrupoles. This relies on the (generally valid) assumption that errors in these quadrupoles are the main source of "β-beating".

This note contains no experimental data or discussion of the measurement device itself. It illustrates theoretical computations with numerical examples and examines the consequences of different ways of extracting the emittance from the beam size. However these have a direct bearing on how measurements—even those performed with perfect instrumentation—are interpreted.

Along the way, I have included a number of technical remarks footnotes which may be useful to people doing optics calculations for LEP at high energy.

2 Calculations

Within MAD [4], there are various ways one *might* go about calculating the emittance and beam size. The following is intended as a review for non-specialists. It sketches the physical meaning of the various methods with a minimum of formalism. Optics afficionados are encouraged to jump straight to Section 3.

2.1 The commands `EMIT` and `ENVELOPE`

The recommended way is to use the command `EMIT` to calculate the emittances and the command `ENVELOPE` to calculate the beam sizes and correlation functions at selected points around the ring. Let me make it clear that, to the best of our present knowledge, *these commands correctly include all physical effects* determining the emittance and beam size in the context of linear single-particle dynamics including synchrotron radiation. I shall therefore use them as a reference and refer to the their results as the "true" beam sizes. "Errors" in other calculations of the beam size or emittance will be quoted with respect to these values.

[1] This note expands and supersedes, but does not invalidate, preliminary calculations shown at the LEP Studies Working Group meeting of 30 April 1996. Those calculations used a crude method to generate vertical emittance from betatron coupling. This note does it better.

If we could put a correct description of the real LEP into MAD, then we would want for nothing but these commands. Unfortunately, we do not know the imperfections of the real machine in sufficient detail and this is where the complications set in.

After finding the closed orbit (fixed point of the one-turn map from the starting azimuth of the ring), MAD finds the eigenvectors of the tangent map and expresses them in terms of the primitive physical coordinates (x, p_x, y, p_y, t, p_t) at the starting point of the ring [4]. These eigenvectors define the three normal modes of linear particle motion about the closed orbit. Integrals around the ring [5] of the mean radiation power and the two-point correlation function of its deviation from the mean (quantum fluctuation) projected into these modes give the radiation damping and diffusion rate from quantum excitation. These are combined to give the emittances of the normal modes. The beam emittances are just the average values of the action variables for each normal mode.

The **ENVELOPE** command expresses the total horizontal displacement at a given azimuth, $x(s)$, say, in terms of the normal modes. It then averages over the distribution to compute the beam sizes $\sigma_x = \sqrt{\langle x^2 \rangle}, \ldots$ from the emittances[2].

Correlation functions such as $\langle xy \rangle$ give the degree of "coupling" between planes.

The optics enters here through the magnitudes of the components of the eigenvectors. The relations in (1) can be viewed as a special case where the horizontal displacement only contains components from the radial "betatron" mode and the synchrotron oscillation mode.

At first sight, the MAD commands do not appear to furnish values of the dispersion functions. This is partly because they are not explicitly needed in the calculations of the beam sizes. In the literature, one finds many slightly different definitions of the dispersion functions.[3] For present purposes, I propose to use a statistical definition

$$\begin{aligned} D_x(s) &= \frac{\langle x(s) p_t(s) \rangle}{\langle p_t(s)^2 \rangle} \\ D_y(s) &= \frac{\langle y(s) p_t(s) \rangle}{\langle p_t(s)^2 \rangle} \end{aligned} \tag{2}$$

The momentum variable p_t is the fractional deviation of the particle's energy from a nominal value (usually given in units of 10^{-3} in MAD). The same values can be derived from the generalised β-functions printed out by MAD's **TWISS3** command which is based on the same physics as **EMIT** and **ENVELOPE**.

In the plots which follow, these dispersion functions will be labelled as "Dx(env)" or something similar, to reflect their derivation from the **ENVELOPE** command.

---

[2] Since the three coordinates (x, y, t) are given in units of metres, the quantity $\sigma_t = \sqrt{\langle t^2 \rangle}$ is just the bunch length. The command **ENVELOPE** shows how it varies around the ring.

[3] Often reflecting the experience and temperament of the author.

2.2 The command TWISS

This command is much deprecated and much used, both with good reason. In the past, its main use was to calculate the Courant-Snyder or “Twiss” functions[4] (including the dispersion function D_x), ignoring the effects of the RF and radiation. In the simplest version of the command, the transverse betatron coupling is also ignored. This command remains of use as a guide to matching new optics (particularly for hadron rings) but you have to be careful what you do with it. In operational practice, the Twiss functions needed in various applications have been taken from a table computed once and for all for an ideal ring. Sometimes better results are obtained by using values measured by the “1000 turns method”.

If the RF cavities and radiation are switched on in MAD, then **TWISS** will calculate Twiss functions taking them into account. In this case, it is well-known that the β-functions are good approximations to those calculated by **TWISS3** but that the dispersion functions are significantly wrong.

3 Examples

The following subsections treat a variety of cases, all based on this year’s nominal physics optics at 82 GeV. The optics is set up using the MAD commands:

```
call "/afs/cern.ch/user/s/slath/public/machines/lep96/lep966.seq"
call "/afs/cern.ch/user/s/slath/public/machines/lep96/dev/10860_physics.lep96"
```

Rather than give further detail here, let me say just say that you can find the files I used in various sub-directories of:

```
/afs/cern.ch/users/j/jowett/public/lep96/beuvbeat/
```

In each case, the optics and beam parameters are calculated for both positrons and electrons. The results of the various MAD commands are dumped out into tables and compared and analysed in a spreadsheet environment.

For technical reasons, and because I want to make direct comparisons of e^+e^-, I show the values of optical functions at the beam position monitors (BPMs) and interaction points, rather than at, say, the source points of the BEUV or other instruments[5].

I used Version 8.19/3 of MAD.

[4] If they are “Twiss”-anything, they are most definitely functions and not “Twiss parameters”.

[5] To allow electrons to circulate in MAD, you have to reverse the sequence of machine elements (time-reversal) and reflect the machine in a mirror to reverse all the magnetic fields (but not the electric fields). This is a consequence of a fundamental physical principle (non-CPT invariance when radiation is included). The BPMs and IPs are elements with zero length and have the agreeable property that, when the sequence is reversed, their entrances and exits are the same and they provide invariant observation points in *all* MAD commands. Indeed, some commands let you specify the centre or either end of an element as an observation point. However not all of the commands I needed to use for this study are so obliging. In fact, the only one which offers this feature *plus* the further convenience of the **SPLIT** command is **OPTICS**. **OPTICS** is physically equivalent to **TWISS**. The **SPLIT** command was introduced a few years ago precisely in order to allow observation points at the source points of instruments like the BEUV which lie inside magnets.

3.1 An Imperfect LEP at 82 GeV

This is an imperfect machine. I added random misalignments and tilts to dipoles, quadrupoles and sextupoles to perturb the optics and generate a moderate amount of betatron coupling. After correcting the RMS horizontal and vertical orbits average orbits[6] down to 0.6 mm and 0.4 mm respectively, the tune was corrected back to its nominal value using the main QF and QD strings. For further details of the magnitudes of the imperfections, please consult the file[7]

`/afs/cern.ch/user/j/jowett/public/lep96/beuvbeat/seed2/opticsim.mad"`

The RF distribution corresponds to the installation of cavities at the 1996 startup with typical maximum voltages. As seen from Figure 1, the voltage distribution is rather asymmetric around the octants.

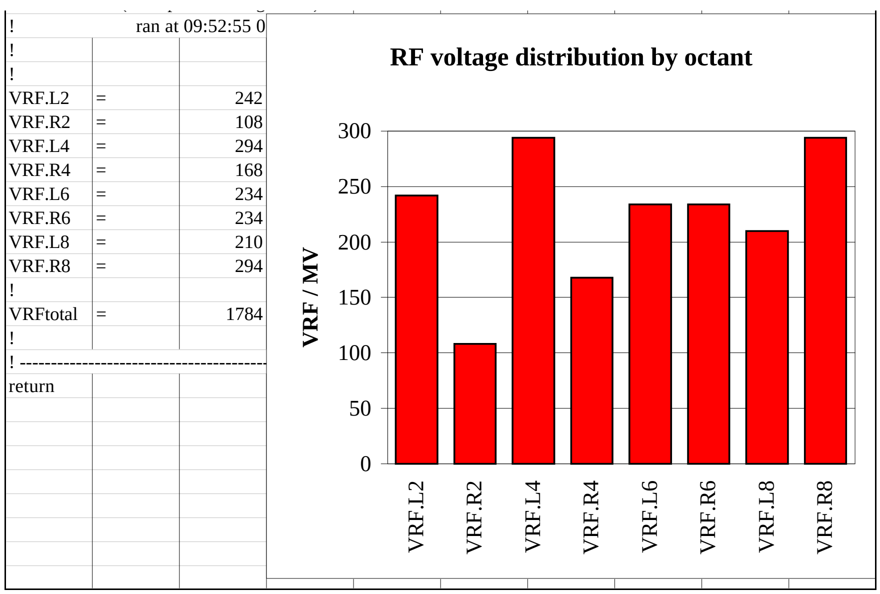

Figure 1: Imperfect ring: Distribution of RF voltage around the ring, computed as described in [7].

The true horizontal and vertical beam sizes of the two beams are shown in Figures 2 and 3. The vertical beam sizes are significantly different because of differences in the optical functions but mainly because the emittances of second normal mode for positrons and electrons are different ($\epsilon_y^+ = 0.3\,\mathrm{nm}$ and $\epsilon_y^- = 0.17\,\mathrm{nm}$). These values correspond to emittance ratios ϵ_y/ϵ_x of 1.1 % and 0.6 %, typical of a very good physics fill.

[6] To be precise, the orbit was corrected with RF and radiation switched off. This corresponds fairly closely to what is done in operations, which is just as well as it is the only way to correct the orbit with the Micado algorithm, even inside MAD.

[7] This is based on an script describing the imperfections of LEP which has evolved over several years. With the improved element selection scheme in the latest version of MAD, it turned out to be necessary to rewrite the script quite substantially.

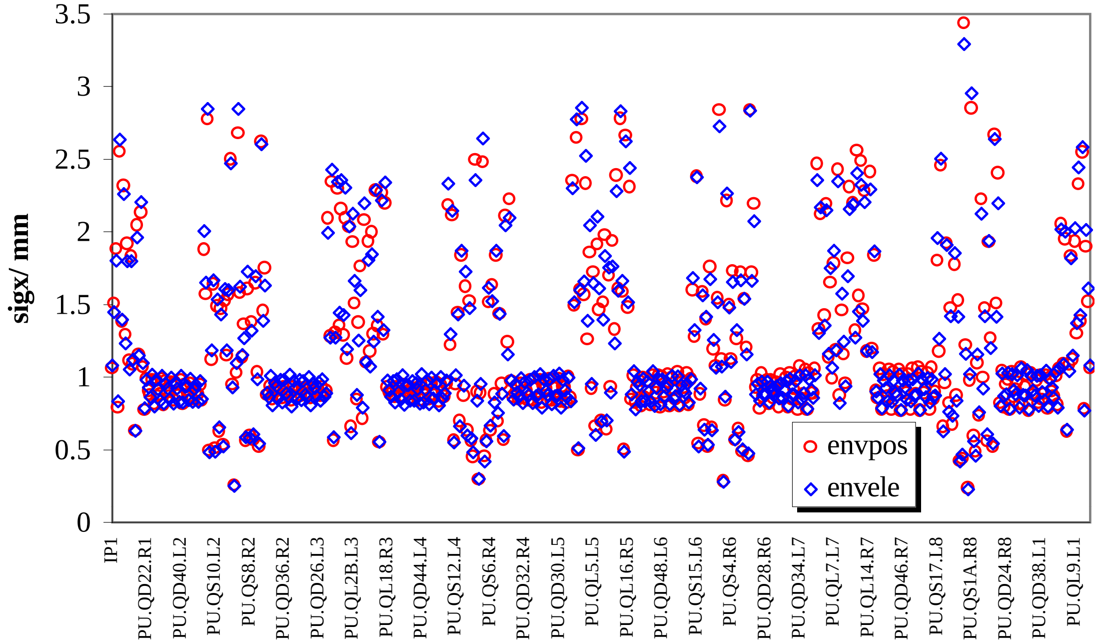

Figure 2: Imperfect ring: The true horizontal beam size (**ENVELOPE** command) at all the BPMs and IPs around the ring for both e^+ and e^-.

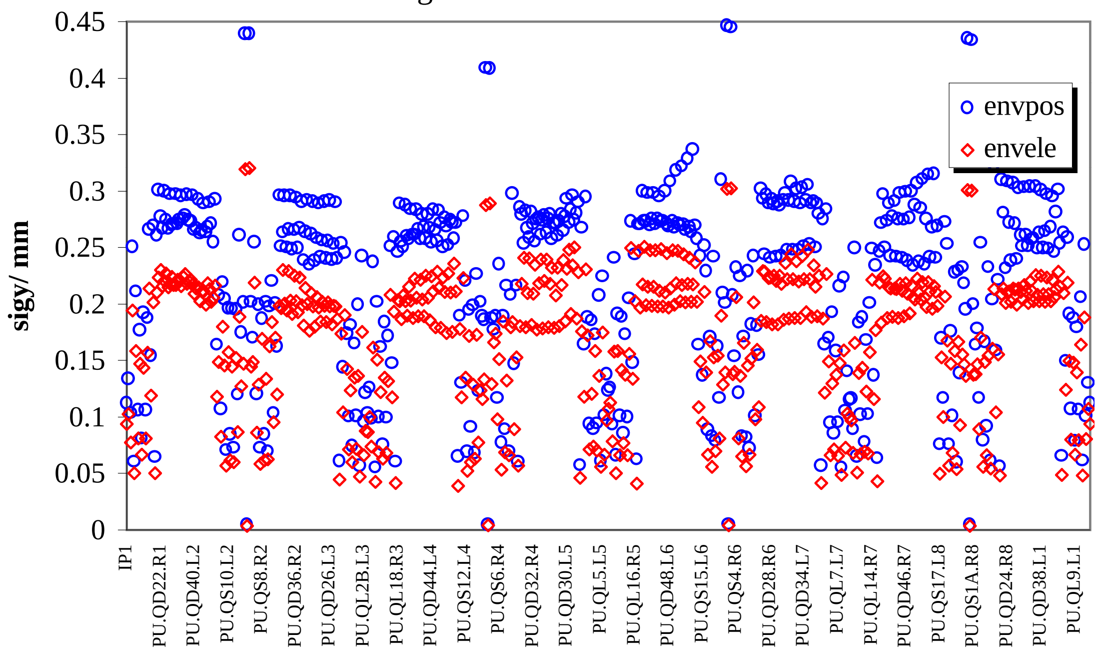

Figure 3: Imperfect ring: The true vertical beam size (**ENVELOPE** command) at all the BPMs and IPs around the ring for both e^+ and e^-.

Figures 4 and 5 show the ratios of the β-functions calculated with and without radiation. The "β-beating due to radiation" is quite significant in the vertical mode and different for the two beams.

Figures 6 and 7 show the differences in beam sizes obtained by naive application of formulas like (1) with various versions of the optical functions. In each case a percentage error relative to the true beam size is quoted. Except for the second case (where the dispersion function computed using the `TWISS` command with radiation is used), the results are reasonable in the horizontal plane. In the vertical plane the errors are large in all cases.

Figure 8 shows that the statistical definition, (2), of the horizontal dispersion function is in rather good agreement with the D_x obtained from the `TWISS` command *without* radiation and RF. On the other hand, Figure 9 shows that the statistical dispersion in the vertical plane is about a factor of two larger in this example.

Figures 6 and 7 showed the error obtained by mis-predicting the beam size from a knowledge of the emittances. It is perhaps more important to consider the process of estimating the emittance from knowledge of the beam sizes, e.g., at a device like the BEUV. For this purpose we can imagine that there is a device measuring the beam size at every BPM or IP and invert (1) to estimate the emittance (assuming knowledge of the energy spread). Figures 10 and 11 shows the resulting error in the estimate of horizontal and vertical emittance. The error is very large in the vertical plane and could easily explain the observed discrepancy between the "measured vertical emittance" and the luminosity monitors[8].

The main reason for the error in the naive calculation is the tilt of the beam profiles in configuration (x, y) space. This is characterised by the quantity

$$R_{xy} = \frac{\langle xy \rangle}{\sqrt{\langle x^2 \rangle \langle y^2 \rangle}} \tag{3}$$

which satisfies $-1 \leq R_{xy} < 1$. This quantity is plotted for the positrons in Figure 12. Despite what would normally be considered a very small global "coupling", this quantity can take on rather large values locally.

The effect of this on the estimation of beam sizes at the IPs is shown (somewhat indirectly) in Figure 13. Here the beam sizes at the IPs are reconstructed from knowledge of the true emittances in various ways. Although there is little scope for error in the horizontal plane, the estimates of vertical beam sizes can vary quite widely. If one takes into account that the vertical emittance being fed into such calculations could in practice be a factor of two or so larger than the true one, it is easy to see that the vertical beam size at the IPs could be over-estimated.

[8] This is not to deny the importance of other effects such as the perturbation of the optics by the beam-beam interaction.

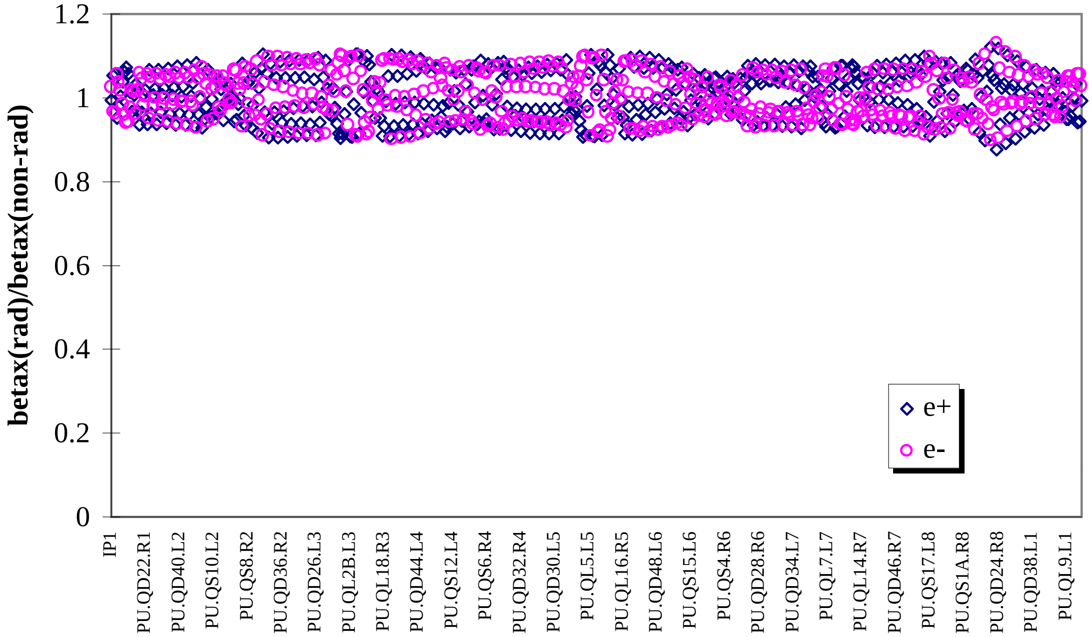

Figure 4: Imperfect ring: The ratio $\beta_x^{\mathtt{rad}}/\beta_x^{\text{non-rad}}$ for both e$^+$ and e$^-$. All calculations include the same imperfections.

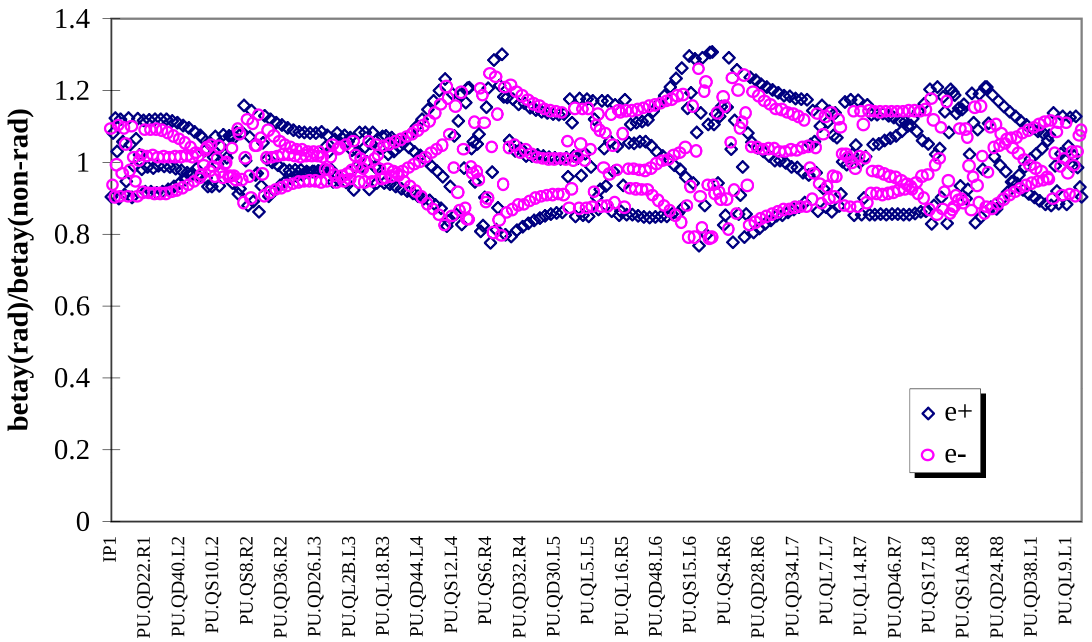

Figure 5: Imperfect ring: The ratio $\beta_y^{\text{rad}}/\beta_y^{\mathtt{non-rad}}$ for both e$^+$ and e$^-$. All calculations include the same imperfections.

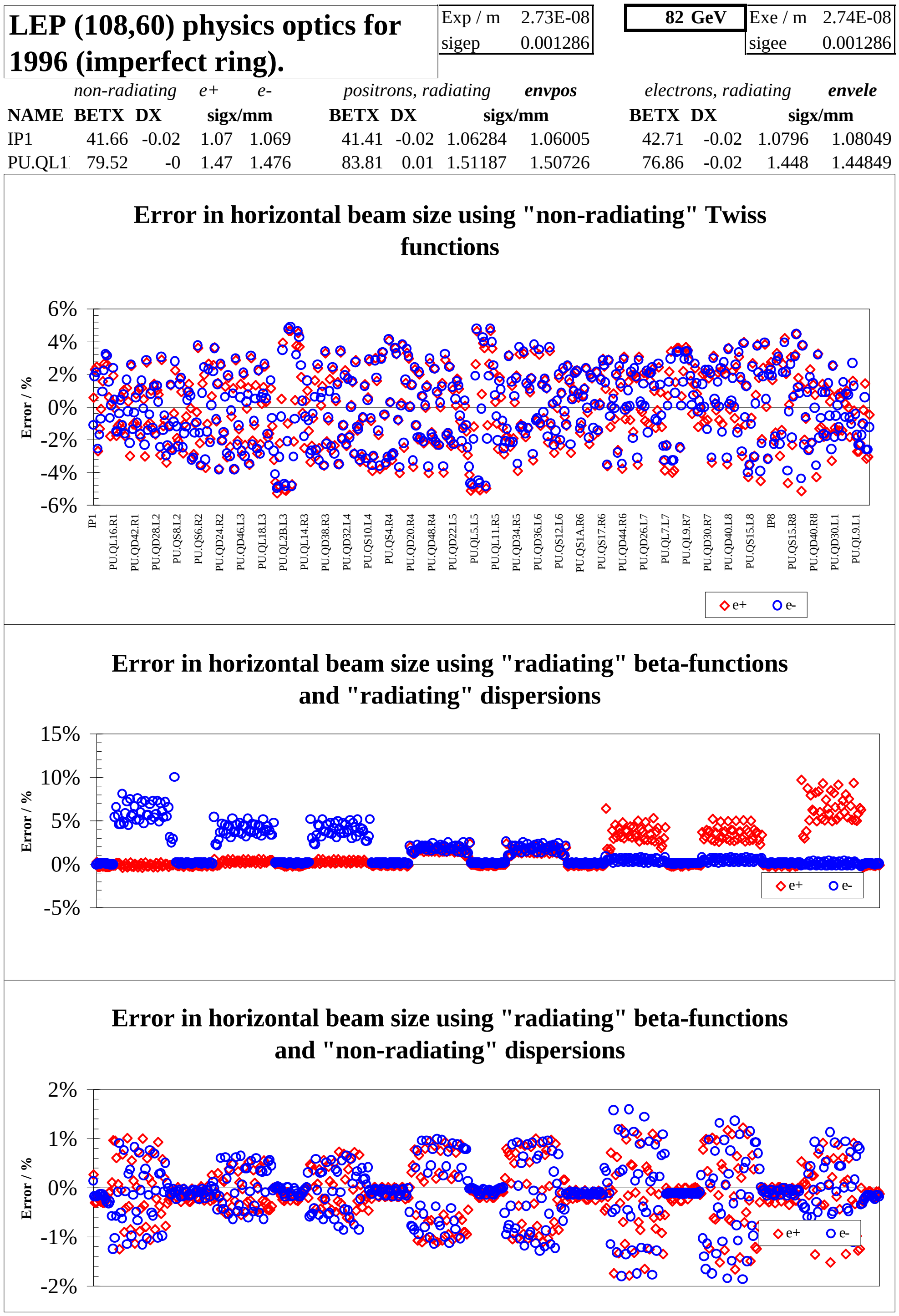

Figure 6: Imperfect ring: These plots show errors in various calculations of the horizontal beam size for both e^+ and e^-, starting from the (true) emittances and constructing the beam sizes according to (1) with various versions of the optical functions. The error is always quoted relative to the true beam sizes calculated by `ENVELOPE`.

LEP (108,60) physics optics for 1996 (imperfect ring).

						Eyp / m	3.01E-10		**82 GeV**	Eye / m	1.7E-10	
						sigep	0.001286			sigee	0.00129	
							1.10%				0.60%	
	non-radiating		*e+*	*e-*	*positrons, radiating*			***envpos***	*electrons, radiating*			***envele***
NAME	**BETY**	**DY**	**sigy/mm**		**BETY**	**DY**	**sigy/mm**		**BETY**	**DY**	**sigy/mm**	
IP1	27.66	0.009	0.091	0.069	25.02	0.01	0.0868	0.112439	30.22	0.01	0.0706	0.09399
PU.QL1	31.78	-0.004	0.098	0.073	34.35	-0.01	0.1017	0.133666	29.76	2E-04	0.0701	0.10228

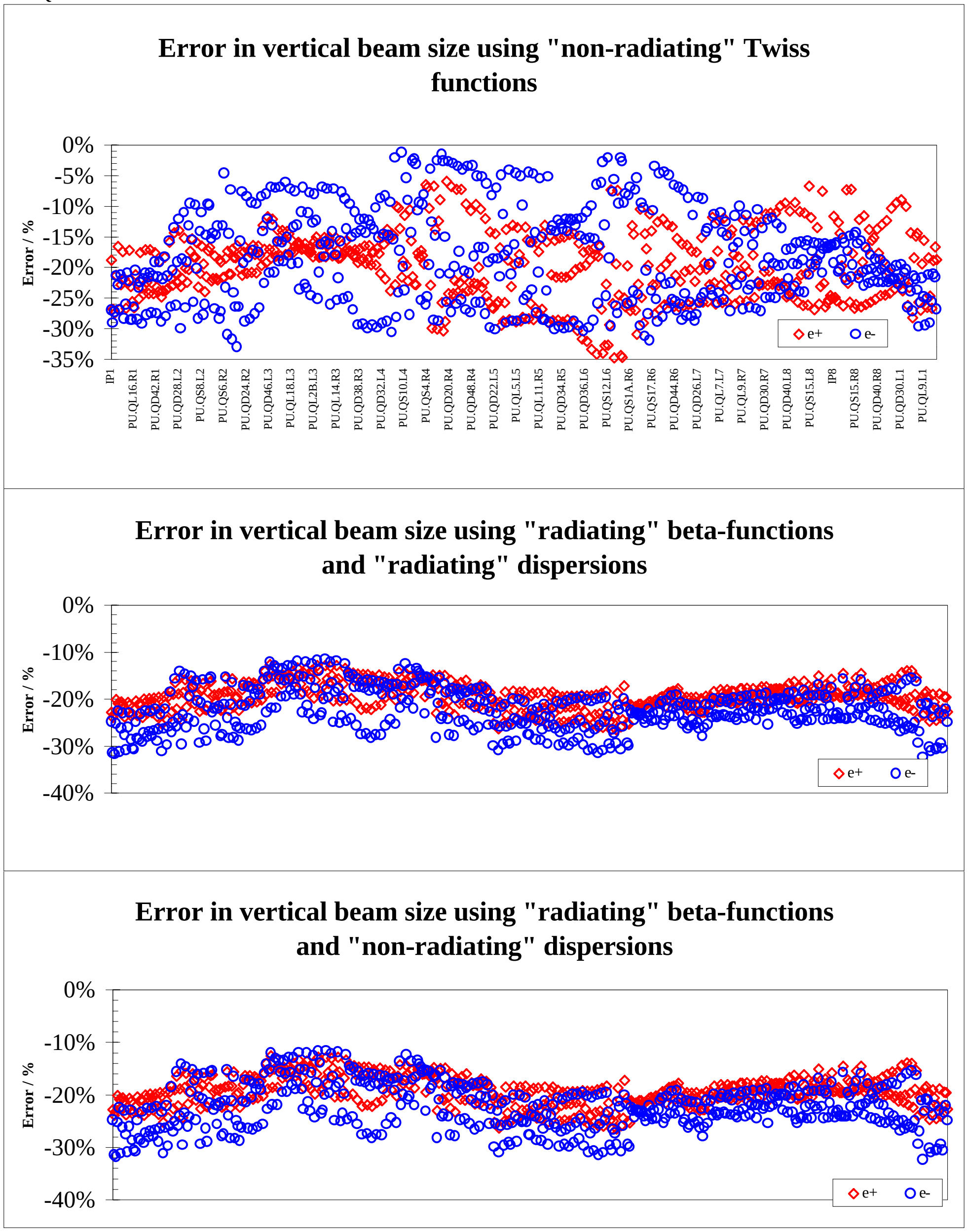

Figure 7: Imperfect ring: These plots show errors in various calculations of the vertical beam size for both e^+ and e^-, starting from the (true) emittances and constructing the beam sizes according to (1) with various versions of the optical functions. The error is always quoted relative to the true beam sizes calculated by `ENVELOPE`.

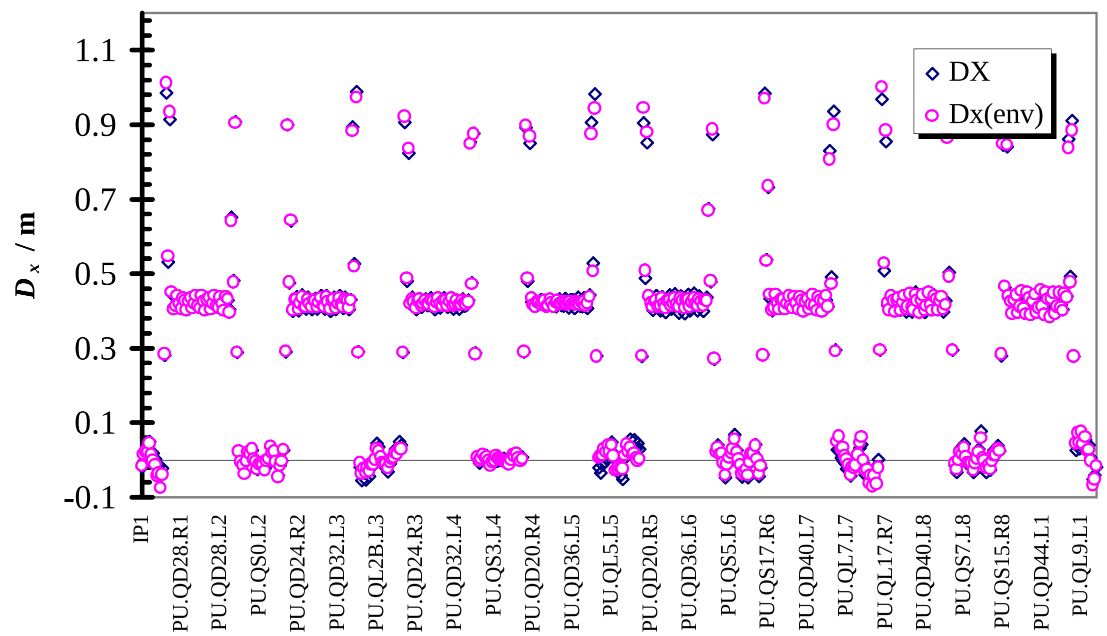

Figure 8: Imperfect ring: Horizontal dispersion function for positrons calculated by the **TWISS** command without radiation compared with that derived from the **ENVELOPE** command using (2).

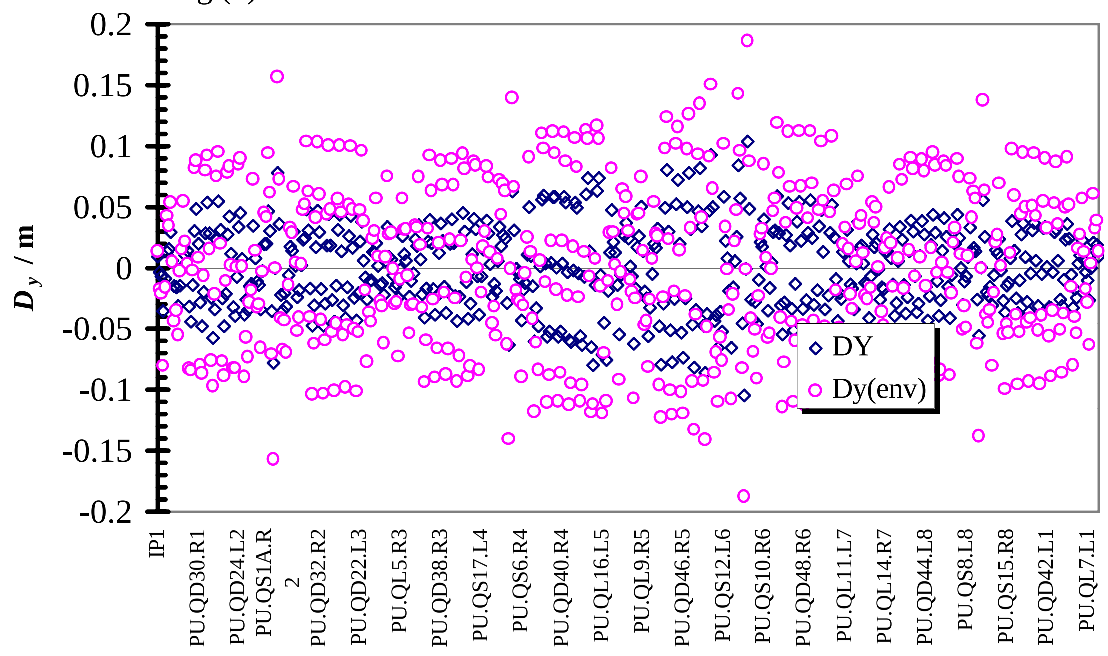

Figure 9: Imperfect ring: Vertical dispersion function for positrons calculated by the **TWISS** command without radiation compared with that derived from the **ENVELOPE** command using (2).

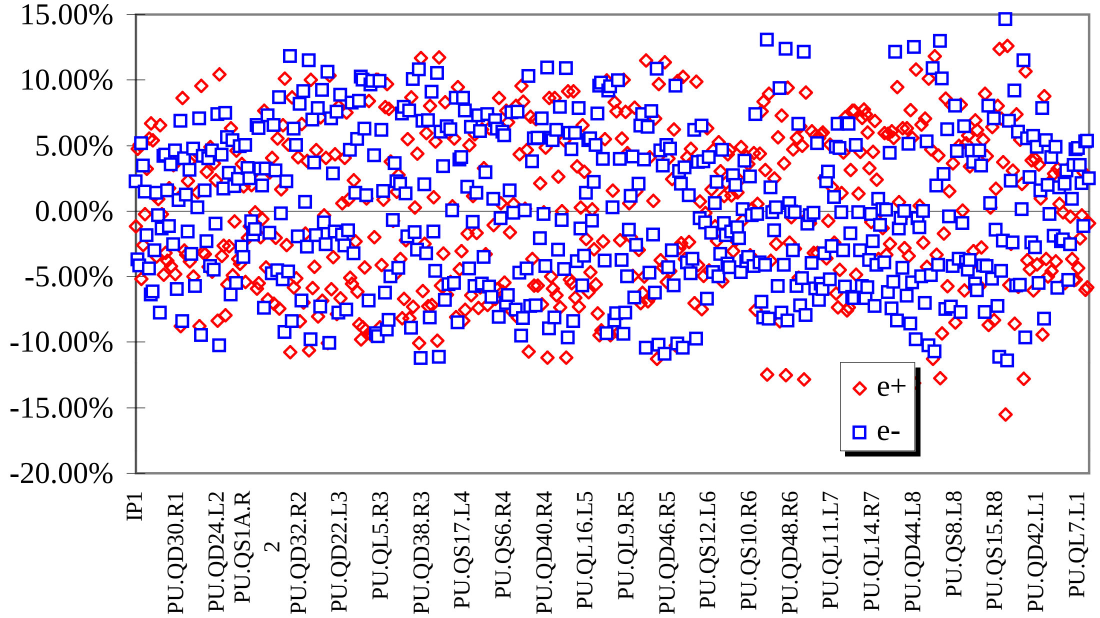

Figure 10: Imperfect ring: The fraction by which the horizontal emittance is overestimated at each BPM when the true beam size (`ENVELOPE`) is converted to an emittance using optical functions calculated by the `TWISS` command without radiation, but with perfect knowledge of the imperfections, according to (1).

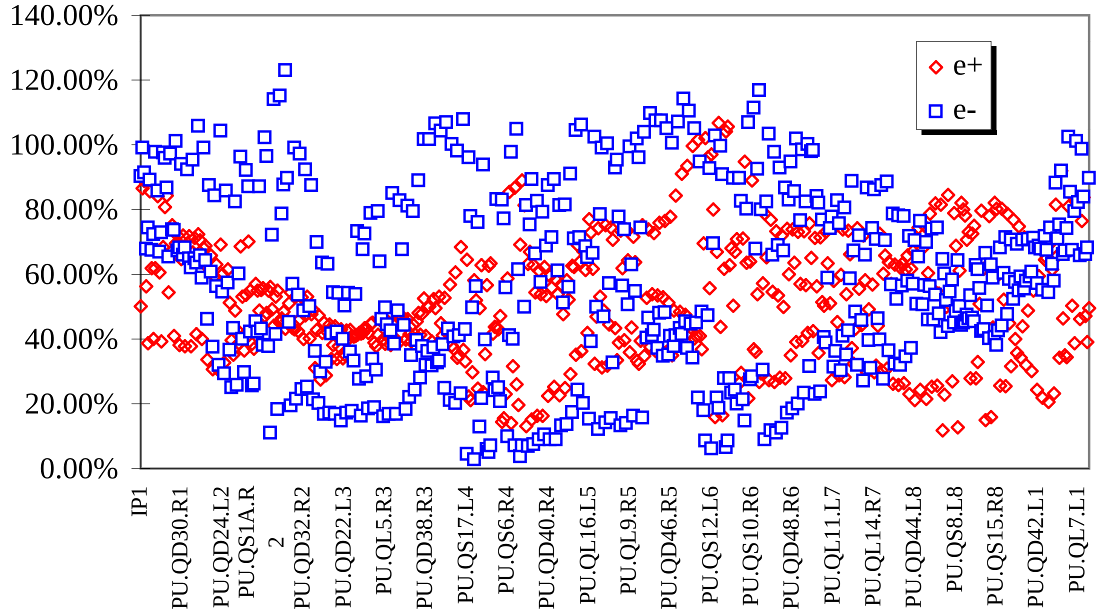

Figure 11: Imperfect ring: The fraction by which the vertical emittance is overestimated at each BPM when the true beam size (`ENVELOPE`) is converted to an emittance using optical functions calculated by the`TWISS` command without radiation, but with perfect knowledge of the imperfections, according to (1).

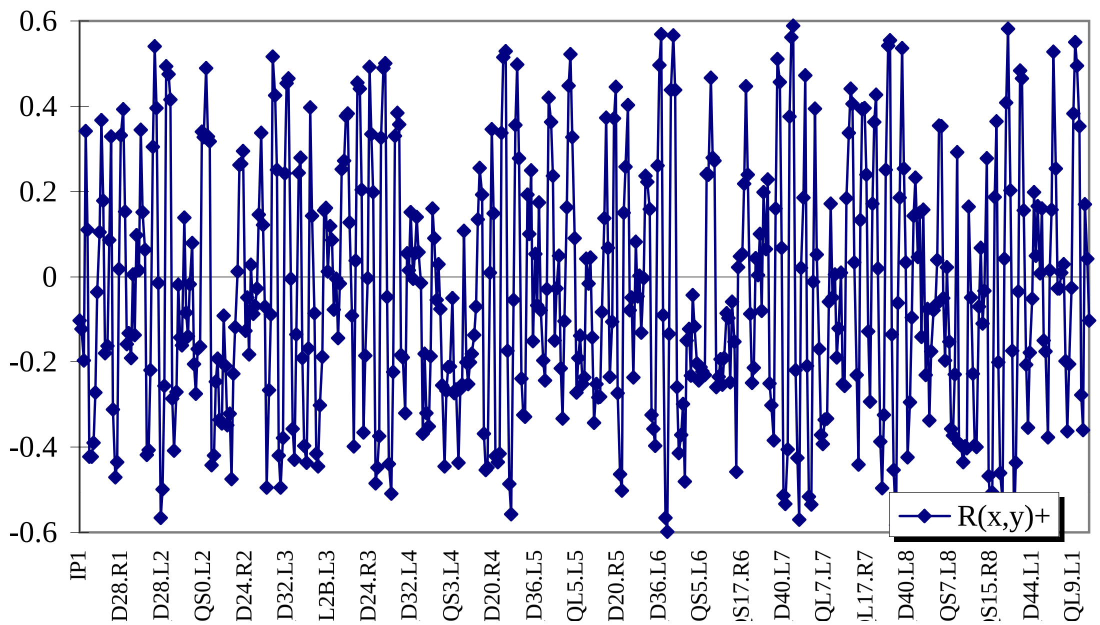

Figure 12: Imperfect ring: The beam tilt for positrons computed by the **ENVELOPE** command in MAD.

Exp / m	2.7E-08	0	82	GeV	Exe / m	2.7E-08
sigep	0.00129	0	0	0 sigee		0.00129
Eyp / m	3E-10				Eye / m	1.7E-10

													mixed functions	
	non-radiating		sigx/mm		positrons, radiating		envpos		electrons, radiating		envele		**sigx / mm**	
NAME	**BETX**	**DX**	**· (non-ra-**	**(non-rad)**	**BETX**	**DX**	**sigx/mm**		**BETX**	**DX**	**sigx/mm**		e+ (mixed)	e- (mixed)
IP2	2.352	-0.0073	0.2535	0.25403	2.35	-0.007	0.25336	0.25331	2.327	-0.006	0.25201	0.25227	0.2533572	0.252626
IP4	3.196	-0.0002	0.2952	0.29587	3.161	7E-04	0.29362	0.29366	3.228	-0.002	0.29672	0.29723	0.2936233	0.297373
IP6	2.872	0.0031	0.2799	0.28052	2.909	0.003	0.28171	0.28156	2.832	0.0029	0.27793	0.27833	0.2817159	0.278547
IP8	1.937	0.0065	0.23	0.23051	2.014	0.004	0.23444	0.23388	1.87	0.0082	0.22608	0.22614	0.2345251	0.226489

	non-radiating		sigy/mm		positrons, radiating		envpos		electrons, radiating		envele		**sigy/mm**	
NAME	**BETY**	**DY**	**· (non-ra-**	**(non-rad)**	**BETY**	**DY**	**sigy/mm**		**BETY**	**DY**	**sigy/mm**		e+ (mixed)	e- (mixed)
IP2	0.044	-0.0002	0.0036	0.00271	0.042	-1E-04	0.00354	0.00445	0.046	-2E-04	0.00275	0.0037	0.0035371	0.002751
IP4	0.048	-0.0005	0.0038	0.00288	0.045	-6E-04	0.00369	0.00459	0.053	-4E-04	0.00296	0.00385	0.0036911	0.002962
IP6	0.052	-0.0007	0.004	0.00306	0.049	-9E-04	0.00384	0.00493	0.059	-5E-04	0.00312	0.00413	0.003838	0.003121
IP8	0.051	0.0001	0.0039	0.00291	0.048	-8E-05	0.00379	0.00444	0.056	0.0002	0.00304	0.0036	0.0037915	0.003035

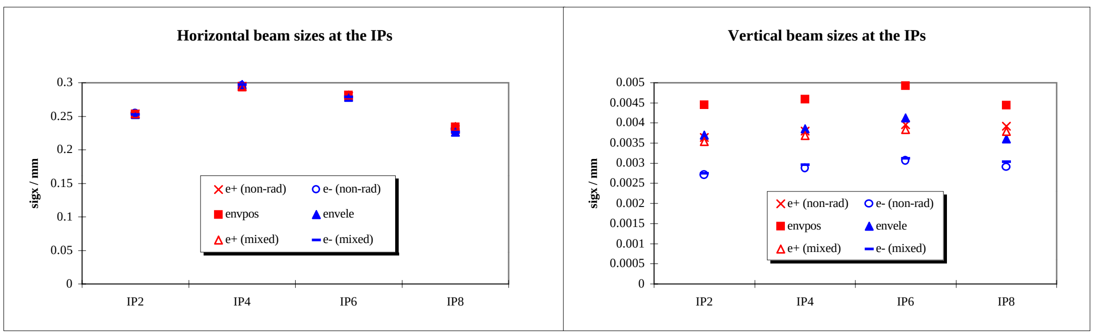

Figure 13: Imperfect ring: A comparison of beam sizes at the IPs computed from the (true) emittances using the non-radiating beam sizes compared with the true beam sizes computed by the **ENVELOPE** command (envpos, envele). Values of the various optical functions at the IPs are also shown in tabular form.

3.2 Effect of a more symmetric RF system

It might be suggested that some of the effects seen in the above example are related to the asymmetry of the RF voltage distribution [6]. To see how much can be attributed to RF asymmetry, I repeated the same calculations with exactly the same machine imperfections but with an RF system adjusted to provide equal voltage in each octant. This meant that certain superconducting RF units were given voltages much higher than they are capable of in reality. The total RF voltage was kept the same as in the example of Section 3.1. The analogous plot to Figure 1 would show eight equal bars of height 222 MV.

Figure 14 shows that symnmstrizing the RF does not get rid of the difference in vertical beam sizes.

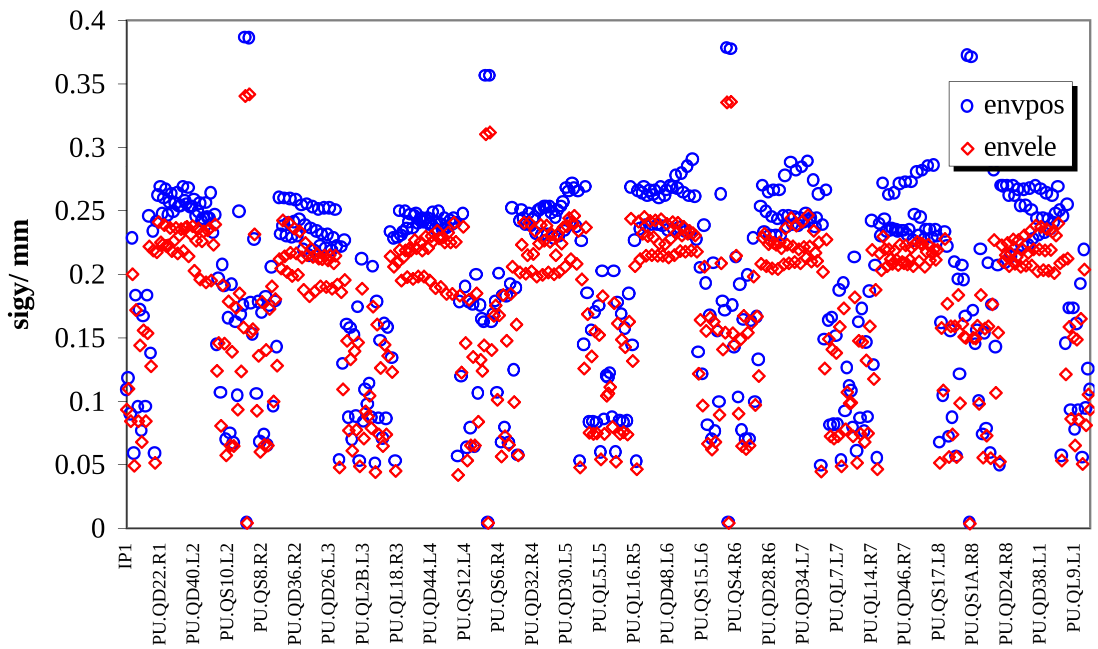

Figure 14: Imperfect ring with symmetric RF: The true vertical beam size (`ENVELOPE` command) at all the BPMs and IPs around the ring for both e^+ and e^-.

Figure 15 shows that it changes, but does not eliminate, the vertical "β-beating due to radiation". The analogues of Figures 4, 5 and 8 (not shown here), are also similar.

Figure 16, the analogue of Figure 9, shows that the difference between the statistical and `TWISS` vertical dispersions remains large. This leads to a an over-estimate of the vertical emittance, Figure 18, comparable to the case with asymmetric RF, Figure 11. Figure 12 shows that the beam tilt is again the main source of the problem.

Figure 19 shows that the beam sizes at the IPs remain different, as in Figure 13, despite the symmetric RF. We can conclude that this is due to imperfections. It seems

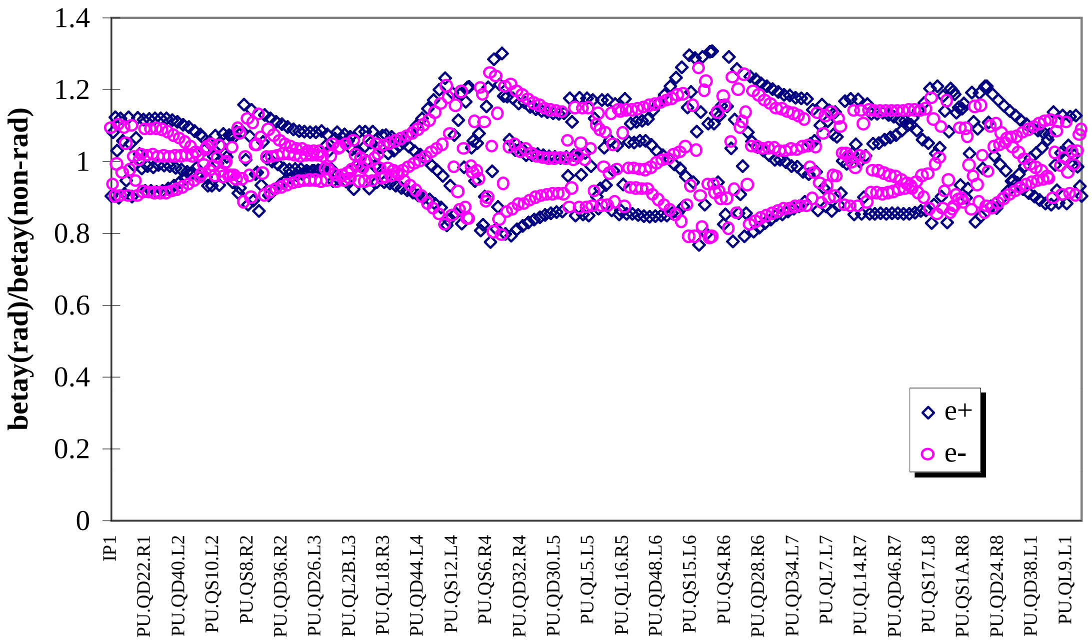

Figure 15: Imperfect ring with symmetric RF: The ratio $\beta_y^{\rm rad}/\beta_y^{\tt non-rad}$ for both e$^+$ and e$^-$. All calculations include the same imperfections.

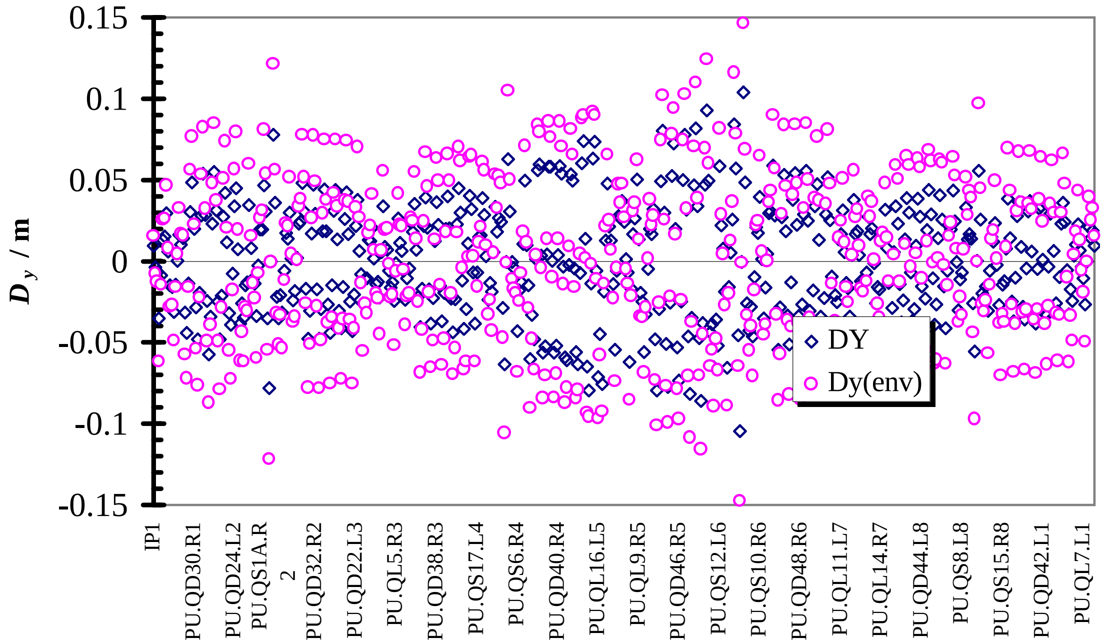

Figure 16: Imperfect ring with symmetric RF: Vertical dispersion function calculated by the `TWISS` command without radiation compared with that derived from the `ENVELOPE` command using (2).

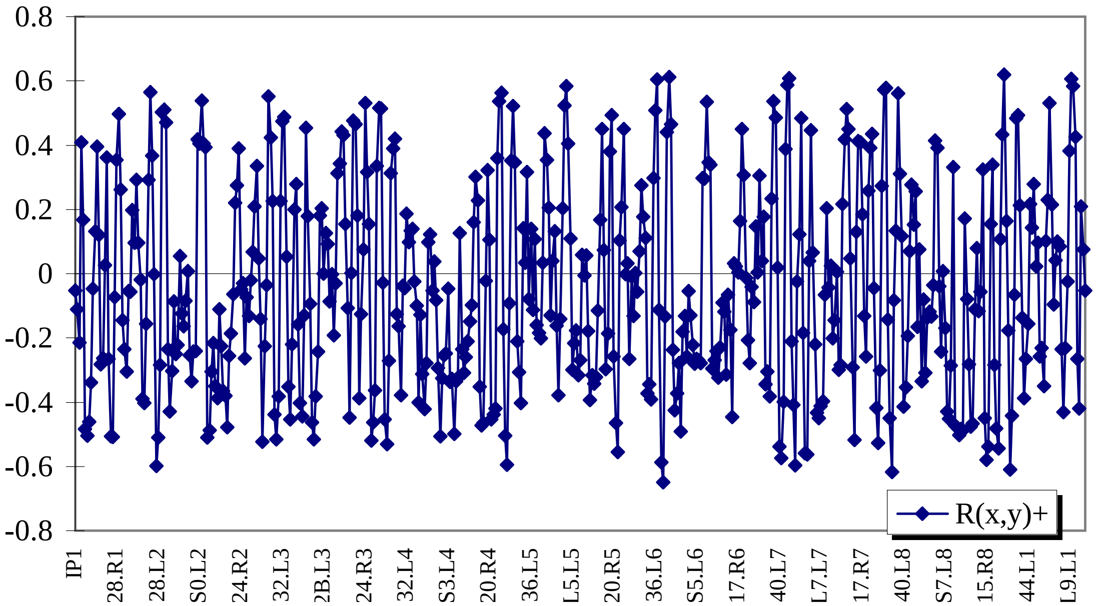

Figure 17: Imperfect ring with symmetric RF: The beam tilt for positrons computed by the **ENVELOPE** command in MAD.

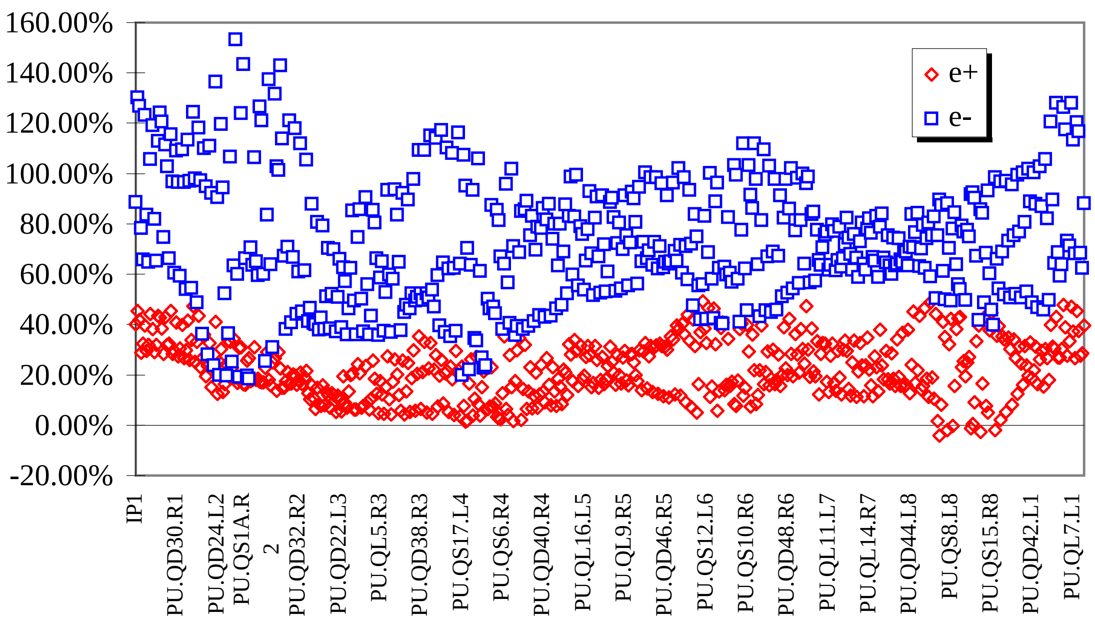

Figure 18: Imperfect ring with symmetric RF: The fraction by which the vertical emittance is over-estimated at each BPM when the true beam size (**ENVELOPE**) is converted to an emittance using optical functions calculated by the**TWISS** command without radiation, but with perfect knowledge of the imperfections, according to (1).

plausible that correction of the β-beating due to imperfections (as in [3]) would help to equalise the beam sizes and luminosity between IPs.

G:\HOME\JOWETT\LEP96\BEUVBEAT\SEED2\OPTICSIN.XLS[Beam sizes at IPs] 21:59

Exp / m	2.7E-08	0	82 GeV		Exe / m	2.7E-08
sigep	0.00129	0	0	0	sigee	0.00129
Eyp / m	3E-10				Eye / m	1.7E-10

	non-radiating		sigx/mm		positrons, radiating		envpos		electrons, radiating		envele		*mixed functions* **sigx / mm**	
NAME	**BETX**	**DX**	**(non-ra-**	**(non-rad)**	**BETX**	**DX**	**sigx/mm**		**BETX**	**DX**	**sigx/mm**		e+ (mixed)	e- (mixed)
IP2	2.352	-0.0073	0.2535	0.25403	2.35	-0.007	0.25336	0.25148	2.327	-0.006	0.25201	0.25553	0.2533572	0.252626
IP4	3.196	-0.0002	0.2952	0.29587	3.161	7E-04	0.29362	0.29342	3.228	-0.002	0.29672	0.297	0.2936233	0.297373
IP6	2.872	0.0031	0.2799	0.28052	2.909	0.003	0.28171	0.28271	2.832	0.0029	0.27793	0.27601	0.2817159	0.278547
IP8	1.937	0.0065	0.23	0.23051	2.014	0.004	0.23444	0.23511	1.87	0.0082	0.22608	0.22478	0.2345251	0.226489

	non-radiating		sigy/mm		positrons, radiating		envpos		electrons, radiating		envele		**sigy/mm**	
NAME	**BETY**	**DY**	**(non-ra-**	**(non-rad)**	**BETY**	**DY**	**sigy/mm**		**BETY**	**DY**	**sigy/mm**		e+ (mixed)	e- (mixed)
IP2	0.044	-0.0002	0.0036	0.00271	0.042	-1E-04	0.00354	0.00418	0.046	-2E-04	0.00275	0.00389	0.0035371	0.002751
IP4	0.048	-0.0005	0.0038	0.00288	0.045	-6E-04	0.00369	0.00425	0.053	-4E-04	0.00296	0.0039	0.0036911	0.002962
IP6	0.052	-0.0007	0.004	0.00306	0.049	-9E-04	0.00384	0.00467	0.059	-5E-04	0.00312	0.00406	0.003838	0.003121
IP8	0.051	0.0001	0.0039	0.00291	0.048	-8E-05	0.00379	0.00429	0.056	0.0002	0.00304	0.00355	0.0037915	0.003035

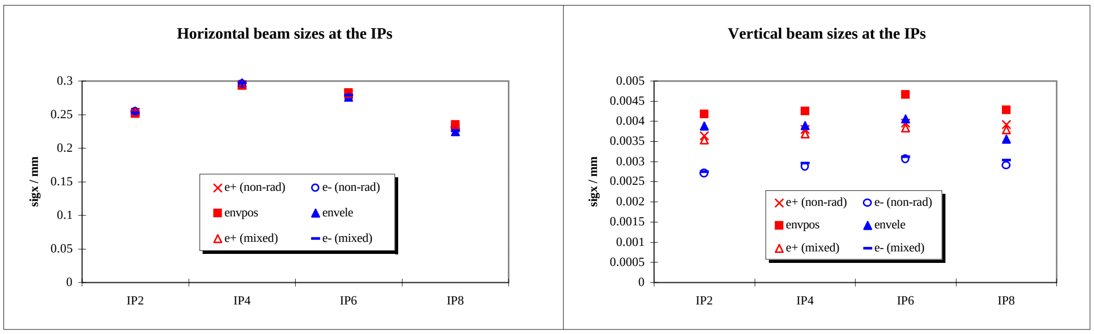

Figure 19: Imperfect ring with symmetric RF: A comparison of beam sizes at the IPs computed from the (true) emittances using the non-radiating beam sizes compared with the true beam sizes computed by the `ENVELOPE` command (envpos, envele). Values of the various optical functions at the IPs are also shown in tabular form.

3.3 Perfect machine with symmetric RF

As a further check on the origin of the effects, this section considers the case of a perfect machine with the same symmetric RF configuration as in Section 3.2. Of course there is now no vertical dispersion or excitation of the vertical emittance so fewer quantities can be compared.

Nevertheless, Figure 20 shows that the horizontal beam sizes are virtually identical for the two beams[9]

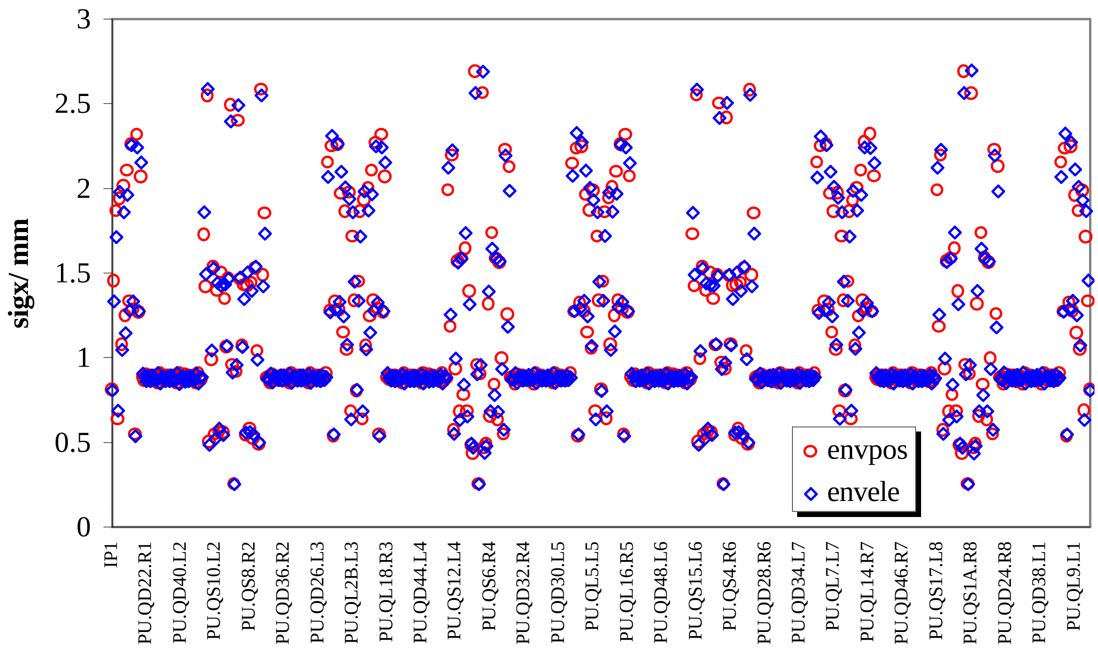

Figure 20: Perfect ring: The true horizontal beam size (**ENVELOPE** command) at all the BPMs and IPs around the ring for both e^+ and e^-.

The "β-beating due to radiation", Figure 21, is now more symmetric but has essentially the same magnitude.

The horizontal dispersion computed from the statistical definition, (2) is in good agreement with that computed by the `TWISS` command without radiation and RF (Figure 22). However the analogue of Figure 10 (not shown) still shows an over-estimate of the horizontal emittance of about 10 %.

The analogue of Figure 13, Figure 23, shows that the horizontal beam sizes are now all equal and predicted well by any method.

[9] I believe, but have not checked in detail, that the small antisymmetries around the IPs are due to the small differences in layout and optics between the interaction regions.

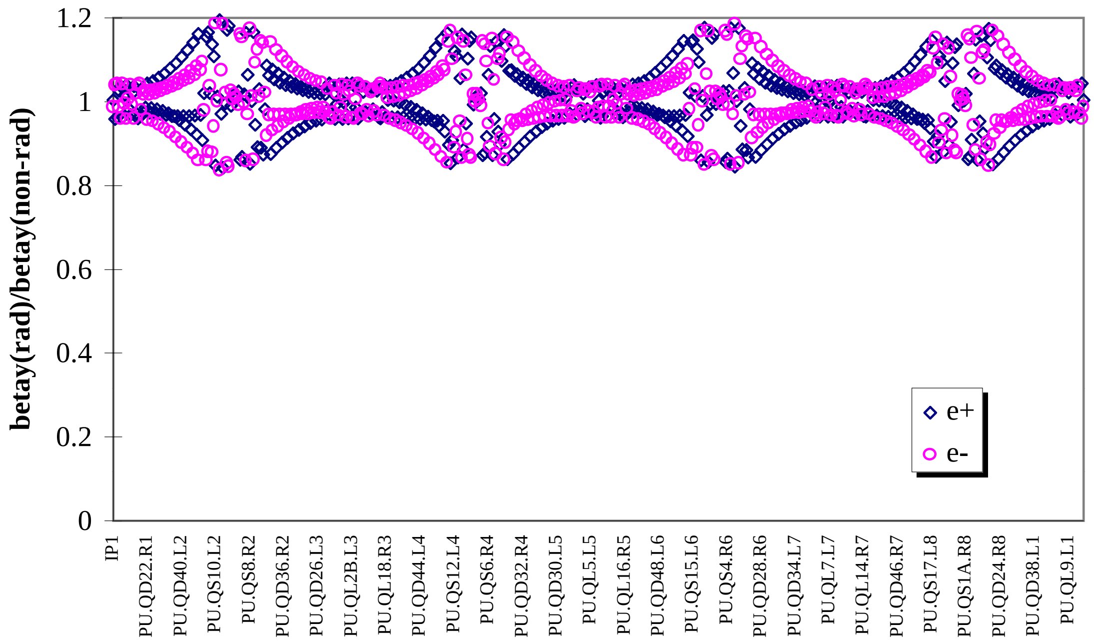

Figure 21: Perfect ring: The ratio $\beta_y^{\mathrm{rad}}/\beta_y^{\mathtt{non-rad}}$ for both e$^+$ and e$^-$. All calculations include the same imperfections.

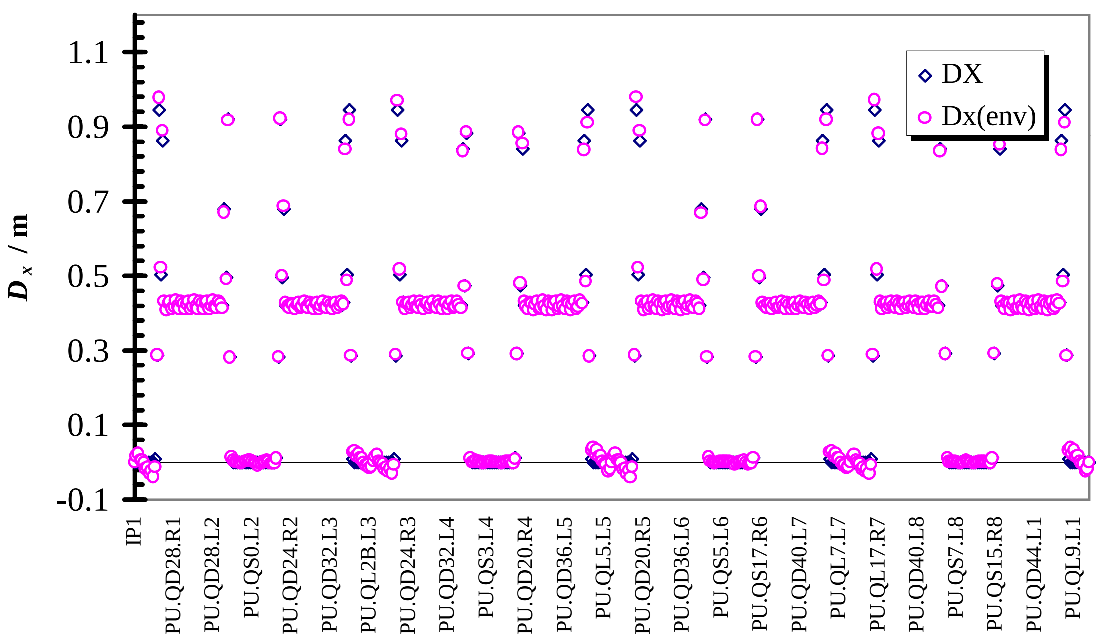

Figure 22: Perfect ring: Horizontal dispersion function calculated by the `TWISS` command without radiation compared with that derived from the `ENVELOPE` command using (2).

Exp / m	2.6E-08	0	82	GeV	Exe / m	2.6E-08
sigep	0.00129	0	0	0	sigee	0.00129
Eyp / m	0				Eye / m	0

	non-radiating		sigx/mm		positrons, radiating		envpos		electrons, radiating		envele		*mixed functions* **sigx / mm**	
NAME	**BETX**	**DX**	**- (non-ra-**	**(non-rad)**	**BETX**	**DX**	**sigx/mm**		**BETX**	**DX**	**sigx/mm**		e+ (mixed)	e- (mixed)
IP2	2.5	-2E-08	0.2526	0.25258	2.495	8E-05	0.25232	0.25232	2.498	0.0006	0.25246	0.25248	0.25232	0.252475
IP4	2.5	2E-08	0.2526	0.25258	2.501	2E-04	0.25262	0.25262	2.5	-8E-04	0.25258	0.2526	0.2526237	0.252596
IP6	2.5	-2E-08	0.2526	0.25258	2.497	-6E-04	0.25239	0.25239	2.492	0.0005	0.25217	0.25219	0.2523847	0.252188
IP8	2.5	1E-08	0.2526	0.25258	2.508	4E-04	0.25294	0.25294	2.494	-1E-04	0.25224	0.25225	0.2529391	0.252253

	non-radiating		sigy/mm		positrons, radiating		envpos		electrons, radiating		envele		**sigy/mm**	
NAME	**BETY**	**DY**	**- (non-ra-**	**(non-rad)**	**BETY**	**DY**	**sigy/mm**		**BETY**	**DY**	**sigy/mm**		e+ (mixed)	e- (mixed)
IP2	0.05	0	0	0	0.051	-2E-27	0	1E-16	0.051	4E-28	0	1.9E-16	0	0
IP4	0.05	0	0	0	0.051	-6E-28	0	1E-16	0.051	-4E-28	0	1.9E-16	0	0
IP6	0.05	0	0	0	0.051	-7E-28	0	1E-16	0.051	-7E-28	0	2E-16	0	0
IP8	0.05	0	0	0	0.051	3E-28	0	1E-16	0.051	-7E-28	0	2E-16	0	0

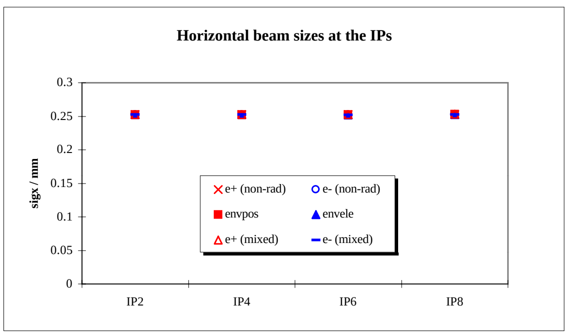

Figure 23: Perfect ring: A comparison of beam sizes at the IPs computed from the (true) emittances using the non-radiating beam sizes compared with the true beam sizes computed by the **`ENVELOPE`** command (envpos, envele). Values of the various optical functions at the IPs are also shown in tabular form.

4 Conclusions

The simulations above illustrate the following points:

- The vertical emittances of positrons and electrons can easily be substantially different at high energy. The differences are *not* due to the asymmetry of the RF voltage distribution but rather to the distribution of the imperfections generating the vertical emittance.

- Similar remarks hold for the optical functions.

- Using optical functions—the dispersion in particular—calculated without the inclusion of radiation can lead to substantial errors in the conversion of a measured *vertical* beam size into a value for the vertical emittance. The same procedure in the horizontal plane gives numerically acceptable results.

- With small values of the emittance ratio, the tilt of the beam ellipse in configuration space can be large enough to confound the attempt to convert a measured vertical beam size directly into an emittance.

- It may be possible to provide a better estimate of the vertical emittance using the value of the correlation function $\langle xy \rangle$ together with the mean-square beam sizes $\langle x^2 \rangle$ and $\langle y^2 \rangle$. The formula

$$\epsilon_y = \frac{\langle x^2 \rangle \langle y^2 \rangle - \langle xy \rangle^2}{\beta_y \langle x^2 \rangle}. \tag{4}$$

  neglects terms describing the coupling of the second mode back into the first and certainly applies only when the dispersion at the BEUV monitors can be neglected. The quantities appearing on the right-hand side are all accessible to measurement. The function β_y should be computed with radiation included (although this is still an approximation). I have tried this with the numerical data described above but, unfortunately, it did not work much better as an estimator of ϵ_y.

It may be objected that the vertical emittances in the imperfect machine considered above were too small, exaggerating the errors and differences between beams. To check this, I repeated the calculations with another seed for the random imperfections and larger amplitudes of the random tilts. The resulting emittances were larger, $\epsilon_y^+ = 5.2\,\mathrm{nm}$ and $\epsilon_y^- = 1.7\,\mathrm{nm}$, but the other results confirmed most of the above conclusions except that the overestimate of vertical emittance from beam size was less, typically 20 %. Application of (4) also worked a little better.

As a final conclusion, I consider that it is worth implementing (4) as an improved method for estimating the vertical emittance in operation. However it cannot be expected to work well for small ϵ_y. If reliable vertical dispersion values are available from measurements they might also be incorporated (together with the computed energy spread) as a correction to this formula.

References

[1] P. Castro et al, “Cross-calibration of emittance measuring instruments in LEP”, CERN SL–MD Note 202 (1996).

[2] M. Lamont, SL Performance Committee meeting of 17 April 1996.

[3] E. Keil, “Off-line simulation of beta-beating correction”, CERN SL Note 96–25(AP) (1996).

[4] H. Grote, F.C. Iselin, “The MAD program (Methodical Accelerator Design) : Version 8.16 ; User’s Reference Manual”, CERN SL 90–13 (AP) rev. 4 (1995). The manual for the version of MAD used in this note is presently available on the World-Wide Web at `http://hpariel.cern.ch/fci/mad/mad.html`

[5] A.W. Chao, “Evaluation of beam distribution parameters in an electron storage ring”, J. Appl. Phys. **50(2)**, 1979

[6] J.M. Jowett, “Problems expected from RF asymmetries”, in J. Poole (Ed.), Proceedings of the Sixth Workshop on LEP Performance, Chamonix, January 1996, CERN SL/96-05 (DI) (1996).

[7] J.M. Jowett, “RF Voltage Distribution in LEP with MAD”, CERN SL Note 96–29(AP) (1996).